

DEFORMATION CHANGES OF CONTROL POINTS

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ABSTRACT

Article serves to become familiar with the theoretical foundation deformation investigation, theory of errors and smoothing. Levelling network can be processed by the method of least squares (LSM) based on regular Gauss - Mark Models. The process consists of geodetic networks processing the measured values in general construction of a theoretical model of deterministic and stochastic, and then a statistical model. The only way to get information about the concept is a direct measurement using an implementation of the vector. The processing of measured elevations in the network was based on the general relationship that are a function of elevation offset the high points and measurements are burdened with actual errors ε . However, in terms of an effective approach in defining the date geodetic network is preferable to use model settlement intermediary measurements with the conditions, which allows a more flexible approach in defining the date geodetic network and also allows examining the stability of reference points within a single process. Date elevation is assigned based on the network purposefully elected, each independent of conditional equations lying G^T on the estimated parameters.

Track extremely slow forms, eg. Lateral movement (mm/year) or tectonic movements of the fault structure was replicated monitoring optical-mechanical device dilatometer TM-71, which is capable of detecting motion and micro-displacement in three directions and also is able to record in the two rotation axes. Facilities should be complete information on the movement of cracks using Moiré, which were successfully implemented precisely measuring instruments for cracks. Moiré effect is mechanical interference that appears in the form of black and white stripes, light induced the upstream grid. Moiré density increases with the density of the grid and grid movements evoke movement strips.

Keywords: the vector of corrections, the configuration matrix, the cofactor matrix, the levelling

INTRODUCTION

Measuring the observed vertical control points in the historic city of Košice was made primarily because the results that emerge from the measurement is to be used in the management and protection of cultural historical monuments of the city of Košice. This article focuses on the implementation of height measurement, which is to determine whether the area occurred elevation changes. To measure the amendment was used digital level Leica DNA 03, in order to achieve the required accuracy that you require the measurement and setting out the prescription.[9]

1. CONDITIONAL SETTLEMENT PROCESSING NETWORK

Within the measurement were targeted 43 elevations and elevation for 29 points, five of which were points of date. On mathematical and statistical interpretation of the observed changes in elevation points was elected a processing program that comes with the required calculation and statistics functionalities and allows the results of vertical displacement levelling points. After verification of the individual sections was necessary to calculate the mean mileage mistake entire levelling strength (

Tab. 1). [9]

Tab. 1 Criteria for precision levelling line

Central kilometre error	m_0	0,39907	[mm]
Extreme deviation Central kilometres errors	m_{0max}	0,50827	[mm]
Mean error levelling line	m_L	1,25598	[mm]
probable error	r	0,26737	[mm]
Central kilometre random error	η	0,40147	[mm]
Systematic error of kilometre	σ	0,90143	[mm]

Central kilometre error

$$m_0 = \frac{1}{2} \sqrt{\frac{1}{n_R} \sum \frac{\rho^2}{R}} \quad (1)$$

where n_R the number of sections.

Extreme deviation Central kilometres errors for levelling network I st order

$$m_{0max} = 0,40 + \frac{1}{\sqrt{n_R}} \quad (2)$$

where n_R the number of sections.

Mean error leveling line

$$m_L = m_0 \sqrt{L} \quad (3)$$

Probable error

$$r = 0,67m_0 \quad (4)$$

Mean error any partition

$$m_R = m_0 \sqrt{R} \quad (5)$$

Central kilometre random error

$$\eta = \frac{1}{2} \sqrt{\frac{\sum \rho^2}{\sum R}} \quad (6)$$

Systematic error kilometre (Lallemand)

$$\sigma = \frac{1}{2} \sqrt{\frac{1}{n_L} \sum \frac{\mu^2}{L^2}} \quad (7)$$

L – length levelling section

n_L - number of sections

μ - amount of elevation differences BACK and FORE

The analysis of the measured data shows that the mean error m_0 kilometre does not exceed the value of the extreme deviation of the mean error kilometres of levelling networks I st order.

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1.1 SETTLEMENT OF NETWORKS LEVELLING

Levelling network can be processed by the least squares method (LSM) based on regular Gauss - Markov Models (GMM):

$$E(\Delta H) = AH; rk(A) = k; V(\Delta H) = s_0^2 P^{-1} \quad (8)$$

$$v = A.d\hat{H} - d\Delta H \dots functional part,$$

$$\sum_{\Delta \hat{H}} = \sigma_0^2 \cdot Q_{\Delta \hat{H}} \dots stochastic part. \quad (9)$$

The process consists of geodetic networks processing the measured values in general construction of a theoretical model of deterministic and stochastic, and then a statistical model. Deterministic model defines the relationship between the theoretical values determined parameters and theoretical values directly measured quantities. Stochastic model, which corresponds to the theoretical model, recognizes that the actual values of measured quantities are not available to our knowledge. The only way to get information about the measured quantity is the direct measurement using an implementation of the vector. [6]

Measurements are burdened with actual errors ε corresponding stochastic model has the form:

$$\begin{aligned} \Delta H &= f(H) + \varepsilon, \\ E[\Delta H] &= f(H), \\ \sigma(\Delta H) &= \sigma(\varepsilon) = \Sigma. \end{aligned} \quad (10)$$

where σ the variance and H are the amount of points leveling network.

If you make development functions $f(\Delta H)$ in the Taylor series neglecting members 2 and higher ranks:

$$f(H) = f(H^\circ) + \left(\frac{\partial f(H)}{\partial H} \right)_{H=H^\circ}, \quad (11)$$

$$dH = f(H^\circ) + AdH,$$

with H° indicate the approximate value of the heights of points, we get the linearized model:

$$f(H^\circ) + AdH = \Delta H + \varepsilon, \quad (12)$$

$$\sum_{\Delta H} = \sigma(\Delta H) = \sigma(\varepsilon),$$

Symbol A denotes the matrix of partial derivations and $d\hat{H}$ refer to as balanced nutrition levels high point. The linearized Gauss - Markov model is the most common solution LSM method, whose basic condition has the form [3]:

$$\Theta = v^T Q_{\Delta H}^{-1} v = \min. \quad (13)$$

After determining the values of balanced supplements heights $d\hat{H}$ it is possible to determine the amount of balanced \hat{H} all aligned points. Adjusted values of elevation $\Delta\hat{H}$ can be determined as the sum of measured elevations ΔH

$$\text{and corrections } v: \Delta\hat{H} = \Delta H + v, \quad (14)$$

Aposteriori variance of unit s_0^2 resulting from the settlement, we consider the best invariant quadratic estimate an unbiased priori variance σ_0^2 calculate the [1] [2] [4] [7]

$$s_0^2 = \frac{v^T Q_{\Delta H}^{-1} v}{r}, \quad (15)$$

where r the number of redundant measurements [8]:

$r = n - k$ - in the case of a binding settlement, n the number of elevation, k number of the establishment of high. [7]

For levelling network was necessary to create matrices, which will enter into the calculation:

H° - vector approximate heights.

ΔH - observational vector:

$$(n,1)\Delta H = [\Delta H_{12} \quad \dots \quad \Delta H_{i,j+1} \quad \dots \quad \Delta H_{n-1,n}]^T \quad (16)$$

ΔH° - vector values approximate elevation

$$(n,1)\Delta H^\circ = [\Delta H_{12}^\circ \quad \dots \quad \Delta H_{i,j+1}^\circ \quad \dots \quad \Delta H_{n-1,n}^\circ]^T$$

$(n,1)\delta\Delta H = \Delta H - \Delta H^\circ$ - vector supplements approximate values

P is a square matrix of weights measured elevations, where the weight p_{ij} elevation ΔH_{ij} is determined by the relationship

$$p_{ij}^{-1} = \frac{s_{\Delta H_{ij}}^2}{s_{01}^2} = \frac{s_{01}^2 L_{ij} [km]}{s_{01}^2} = L_{ij} [km], \quad (17)$$

where L_{ij} the length of the leveling section between adjacent points i, j .

$$A_{n,k} = \begin{bmatrix} \left(\frac{\partial \Delta H_{12}}{\partial H_1} \right)_0 & \dots & \left(\frac{\partial \Delta H_{12}}{\partial H_k} \right)_0 \\ \vdots & & \vdots \\ \left(\frac{\partial \Delta H_{i,j}}{\partial H_1} \right)_0 & \dots & \left(\frac{\partial \Delta H_{i,j}}{\partial H_k} \right)_0 \\ \vdots & & \vdots \end{bmatrix}$$

$(n, k)A$ - configuration matrix levelling network whose coefficients are developed on the basis of model equations for elevation [7]

1.2 DEFINING THE DATE LEVELLING NETWORKS

In terms of efficient approach for defining the date geodetic network is preferable to use model settlement intermediary measurements with the conditions, which allows a more flexible approach in defining the date geodetic network and also allows examining the stability of reference points within a single process. Date elevation is assigned a network based on the purposeful elected, each independent of conditional equations G^T lying on the estimated parameters in the form:

$$G^T = \begin{Bmatrix} g_1(H_1, \dots, H_k) \\ \vdots \\ g_p(H_1, \dots, H_k) \end{Bmatrix} \quad (18)$$

under which GMM relationship (9) enter into the following shape:

$$\begin{aligned} v &= A.d\hat{H} - d\Delta H \\ \Theta &= G^T .d\hat{H} + h \end{aligned} \quad (19)$$

$$\sum_{\Delta\hat{H}} = \sigma_0^2 \cdot Q_{\Delta\hat{H}}$$

for balanced amount of points we can write

$$\hat{H} = H^o + d\hat{H}, \tag{20}$$

where estimates supplements offset the high defines the relationship

$$d\hat{H} = Q_{\hat{H}} \cdot A^T \cdot Q_{\Delta H}^{-1} \cdot d\Delta H. \tag{21}$$

Vector residues v_i for each elevation ΔH is determined by the relationship

$${}_{(n,1)}v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = A \cdot d\hat{H} - \delta\Delta H. \tag{22}$$

Offset the high dispersion lie on the diagonal covariance matrix

$$\Sigma_{\hat{H}} = s_o^2 Q_{\hat{H}}, \tag{23}$$

where the apriori variance of unit s_o^2 of model for m unknown, k the parameters a p independent equations of conditional commitment on the estimated parameters defines the relationship

$$s_o^2 = \frac{v^T \cdot Q_{\Delta H}^{-1} \cdot v}{n - k + p}. \tag{24}$$

Accuracy characteristics of liquidating elevations determined by their covariance matrix

$$\Sigma_{\Delta\hat{H}} = s_o^2 Q_{\hat{H}} = s_o^2 (A \cdot Q_{\hat{H}} \cdot A^T) \tag{25}$$

[7] [8]

1.3 VALIDATION OF THE MODEL LEVELLING SETTLEMENT NETWORK

To assess the correctness and complexity estimator model (19) was used to test the global test statistic

$$T = \frac{v^T \cdot P \cdot v}{\sigma_0^2} \approx \chi_\alpha^2(f), \tag{26}$$

from which we can derive the critical value of the

$$\text{ratio } \frac{s_o^2}{\sigma_0^2} < \frac{\chi_\alpha^2(f)}{f}, f = n - k + p \text{ degrees of}$$

freedom, $s_o = 1,30$ and significance $\alpha = 0,05$, spits it tattooed ratio of prior and posterior unit

variance $T = \frac{s_o^2}{\sigma_0^2}$ is smaller than the critical

$$\text{value of the global test } T_{krit} = \frac{\chi_\alpha^2(f)}{f} = 1,59.$$

With 95% likely to accept the null hypothesis that the model chosen is correct, complete and corresponds to reality.[4]

After leveling network further testing has been performed remote measurements based on testing of their respective repairs. Was chosen test statistic (T_i) Pope τ test, which is used for detection of outliers contained in the set of test data, i.e. tested the null hypothesis $H_0 : E(v_i) = 0$ the chosen significance level α .

$$T_i = \frac{|v_i|}{s_{v_i}} = \frac{|v_i|}{s_o \sqrt{|q_{v_i}|}} \approx \tau_{(f)}, \tag{27}$$

where τ the value of the statistical τ distribution.

$$\tau_{(f)} = \frac{\sqrt{f} \cdot t_{(f-1)}}{\sqrt{f - 1 + t_{(f-1)}^2}}, \tag{28}$$

Test statistic was expressed T_i for each of the corrections (27). Subsequently calculated critical value $T_k = \tau_{\alpha/2}(f) = 2,93$ (28). After testing the null hypothesis, we can say that the null hypothesis is adopted as a risk α can be tested the amounts repair all measurements considered statistically significant.

1.4 STATISTICAL EVALUATION ALTITUDE CHANGE POINT

The elevation change (**Error! Not a valid bookmark self-reference.**) observed point we can think with some probability P (%) when the vertical displacement p' between two stages of measurement is at least two to three times greater than the accuracy of the measurement standard deviation expressed $s_{\bar{p}}$, [1]

$$\bar{p} \geq t s_{\bar{p}}, \tag{29}$$

where t confidence is a factor ($t = 2,0$ or $2,5$) according to the chosen level of significance α (0,05 or 0,01).

$${}^{(1,2)}\bar{p}_i = {}^{(2)}\hat{H}_i - {}^{(1)}\hat{H}_i \quad (30)$$

$$s_{p_i}^2 = {}^{(2)}s_{\hat{H}_i}^2 + {}^{(1)}s_{\hat{H}_i}^2 \quad (32)$$

$$s_{p_i}^2 = \left(\frac{\partial \left({}^{(2)}P_i \right)}{\partial \left({}^{(2)}\hat{H}_i \right)} \right)^2 \cdot {}^{(2)}s_{\hat{H}_i}^2 + \left(\frac{\partial \left({}^{(1)}P_i \right)}{\partial \left({}^{(1)}\hat{H}_i \right)} \right)^2 \cdot {}^{(1)}s_{\hat{H}_i}^2 \quad (31)$$

Tab. 2 Height changes and their statistical assessment

Point number	Epoch 1995		Epoch 2010		Elevat. change	Stand. devia.	Acceptance field	Statisti.. significt. elevat. change	points	area
	H	s _H	H	s _H	ΔH _{1995,2010}	s _{ΔH}	t*s _{ΔH}	ΔH > t*s _{ΔH}		
	[m]	[mm]	[m]	[mm]	[mm]	[mm]	[mm]	< ! >		
ZNSKE 19-500	218,90651	0,33	218,90651	0,00	0,000	0,33	0,66		datum	Datum area North
ZNSKE 17-505	213,82717	0,45	213,82717	0,00	0,000	0,45	0,90			
ZNSKE 17-506	217,16367	0,38	217,16367	0,49	0,000	0,62	1,24			
ZNSKE 17-504	211,85619	0,46	211,85614	0,81	-0,046	0,93	1,86			
ZNSKE 17-503	210,89894	0,43	210,89889	0,97	-0,050	1,06	2,12			
JK-505	210,58519	0,36	210,58508	1,07	-0,114	1,13	2,26			
ZNSKE 17-501	210,09470	0,36	210,09464	1,06	-0,065	1,12	2,24	observed	Observed area	
JK-504	209,81527	0,35	209,81506	1,10	-0,210	1,15	2,31			
JK-503	209,03549	0,32	209,03519	1,17	-0,296	1,21	2,43			
JK-502	208,86388	0,29	208,86356	1,20	-0,323	1,23	2,47			
JK-501	208,87243	0,24	208,87209	1,22	-0,341	1,24	2,49			
ZNSKE 13-502	209,08620	0,40	209,08498	1,28	-1,218	1,34	2,68			
ZNSKE 10-501	208,45698	0,21	208,45624	1,27	-0,738	1,29	2,57			
KZL-500	208,36450	0,00	208,36401	1,24	-0,488	1,24	2,48			
JZL-560	208,41932	0,23	208,41873	1,25	-0,594	1,27	2,54			
JZL-559	207,67580	0,40	207,67517	1,26	-0,633	1,32	2,65			
ZNSKE 10-503	207,35551	0,37	207,35488	1,34	-0,634	1,39	2,78			
JZL-558	206,21090	0,36	206,21023	1,25	-0,669	1,30	2,60			
JZL-557	205,20869	0,44	205,20807	1,21	-0,620	1,29	2,58			
JZL-556	205,79517	0,47	205,79422	1,18	-0,955	1,27	2,54			
JZL-555	204,32870	0,52	204,32888	1,02	0,177	1,14	2,29			
JZL-554	202,86332	0,54	202,86332	0,80	0,004	0,97	1,93	Datum area South		

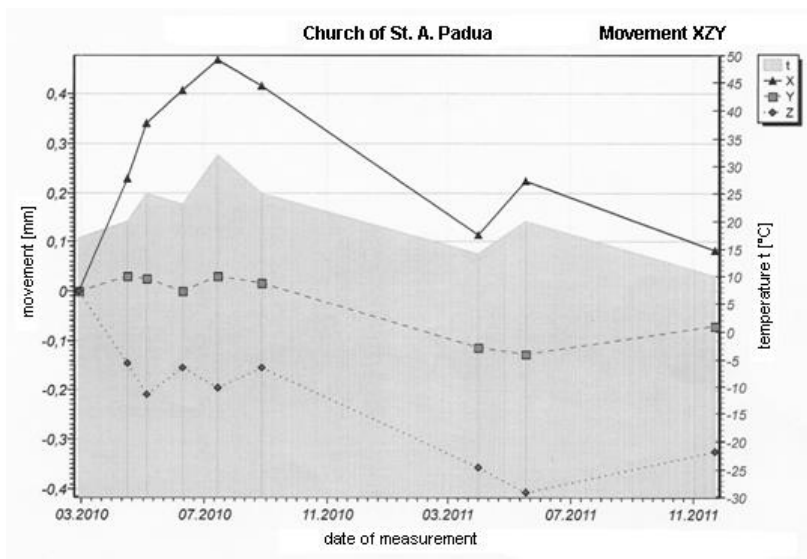
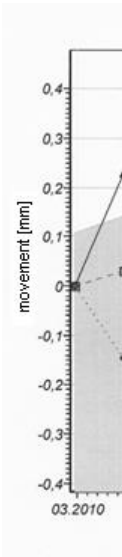
JZL-553	203,41381	0,55	203,41379	0,59	-0,016	0,81	1,61		datum
JZL-552	203,72723	0,55	203,72716	0,26	-0,066	0,61	1,22		
ZNSKE 11-501	205,70707	0,56	205,70702	0,27	-0,050	0,62	1,24		
JZL-546	202,82400	0,57	202,82369	0,33	-0,313	0,66	1,32		
JZL-549	209,07193	0,55	209,07193	0,00	0,000	0,55	1,10		
JZL-548	209,04126	0,55	209,04126	0,00	0,000	0,55	1,10		
JZL-547	207,30425	0,55	207,30425	0,00	0,000	0,55	1,10		

level of significance min. -1,22
 t(α)= 2,0 max. 0,18

2. MEASUREMENT CRACKS IN SIDING HISTORIC BUILDINGS

Dilatometer TM-71 was installed in the loft area church. A. Padua on Main Street. As seen on

Graph 1, maximum value on the axis X = 0,48 mm. Despite the relatively short period of monitoring has been identified distinct and progressive downward trend in one part of the building (axis Z). Although there has been a moderation in the end this is a significant value in terms of future development. Measurements confirmed a slight trend movement in the Y direction, which is interpreted as shearing motion along the crack. The total value represents a movement 0,070 – 0,126 mm. The results confirm previous measurements of current relevant uneven subsidence of the to-progressive.



Graph 1 Graphical representation of spatial kinematic changes cracks in the circumferential wall of church. A. Padua. Movements axes X, Y, Z

CONCLUSION

I believe that I managed to limited space made as comprehensive a theoretical explanation of the matter on the agenda, which is closely linked to practical activities leading to the achievement of the objective of the mathematical - statistical treatment of height measurements followed by their interpretation.

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