LONG-MEMORY PRESENCE IN THE VOLATILITY OF FAT OX DAILY PRICES IN BRAZIL

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ABSTRACT

The major objective of this paper was to examine the long run volatility dependence of fat ox daily price returns in Brazil. The data used in the analysis for the State of São Paulo, Brazil, goes from January 2nd, 2009 to December 30rd, 2013. The methodology employed was based on the ARFIMA - FIGARCH Model, able of detecting the presence of long-memory. The results shows that the ARFIMA (1.d.0) - FIGARCH (1.d.0) Model with a student’s t asymmetric distribution, indicating a fractional long-memory process, high persistence and a hyperbolic decay, with a stationary (reversible) character in the mean as well as in the variance, was the model that best adapted to the data used in the analysis. The fat ox market situation in the period presents strong evidence of corroborating with the obtained results, since a shock in supply yields increases in the commodity price levels. The empirical results suggest the utilization of the proper strategic instruments of hedging in view of the accentuated shock persistence in the fat ox price volatility returns.

Key words: fat ox, price volatility, ARFIMA-FIGARCH Model, long-memory.

1. INTRODUCTION

The fat ox is one of the most important markets in the Brazilian agribusiness. In 2012, 9.2 million tons of beef was produced and 1.4 million tons was exported from Brazil, representing an increase of 2 and 4% from previous year, respectively (USDA, 2014). According to Brazilian Government¹, the Brazilian beef export in the first quarter of 2013 had a better performance than in the same period of the previous year. In the same period, the major beef importers from Brazil were: Russia, Hong Kong, Venezuela, Chile, Egypt, Iran, Italy, Israel, Nederland, and Libya, that all together represented 87.2% of total imports.

The financial crisis that begun in 2007 stimulated market volatility, increasing risks in commodities prices. Additionally, the periodic occurrence of FMD² and the beef price formation process also impacted in production, resulting in an increasing price risk for all market channel agents.

Volatility can be expressed by oscillation movements of commodity prices traded in the Future and Commodities Trade Market. More price volatility results in a greater risk involved and periods of higher volatility tend to be more persistent than periods of lower volatility.

The main objective of this paper was to empirically evaluate the long memory volatility of fat ox price returns in the State of São Paulo, Brazil, describing it as an integrated fractional process in its mean as well as in its variance. Hence, an ARFIMA-FIGARCH model able to detect short run memory as well as long run memory was estimated.

2. A BRIEF LITERATURE REVIEW

Stationary time series with short or long-memory featured by autoregressive fractional integrated mean average (ARFIMA) models has been used and developed for at least two decades (Granger, 1980; Hosking, 1981). In recent years, the interest

¹ Secretaria de Comércio Exterior (Secex).
² Known in Portuguese as febre aftosa. FMD – Foot and Mouth Disease is characterized by fever and blister-like sores on the tongue and lips, in the mouth, on the teats and between the hooves. The disease causes severe production losses and while the majority of affected animals recover, the disease often leaves them weakened and debilitated.
and use of these models has increased considerably (Sowell, 1992; Pai & Ravishanker, 1996; Koop, Ley, Osiewalski, & Steel, 1997; Chan & Palma, 1998).

The ARFIMA models are best used to capture the strong dependence existing among far away observations. The study done by Granger & Joyeux (1980) is known as the first to use this type of model, while the study done by Hosking (1981) is known to present the Box & Jenkins (1976) formal generalized ARIMA (p.d.q) Model for the first time. According to Sowell (1992), the maximum likelihood estimation seems to be the most efficient procedure for the ARFIMA models.

A study done by Macheshchandra (2012) examined the existence of long memory in the Indian stock market, using ARFIMA-FIGARCH models. The ARFIMA Model showed absence of long memory in return series; however, the FIGARCH Model indicated strong evidence of long memory in the volatility of stock returns.

Kilic (2004) using a FIGARCH model concluded that daily returns from the Turkey Stock Exchange Board were not characterized by long memory. Simões et al. (2012) evaluated the prediction capacity of some types of volatility modeling, as well as tested the relevance of some potentially contained information in high frequency data used in a soybeans price series traded globally in the Chicago Board of Trade (CBOT). With that purpose, GARCH and FIGARCH models were compared and adjusted for soybeans series returns, along with autoregressive fractional integrated mean average (ARFIMA) model, adjusted to realize volatility series from existing intra-daily data.

3. METHODOLOGY AND DATA

3.1 The Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski, Phillips, Schmidt & Shin (KPSS) Tests

In order to test the stationarity series, the Augmented Dickey-Fuller - ADF test\(^4\) (1979) was used to verified the integration order of the interested variables, so that the existence or not of unity roots in the temporal series could be verified. The Augmented Dickey-Fuller - ADF test consists on the estimation of the following equation using the Method of Least Square (MLS):

\[
\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta Y_{t-i} + \varepsilon_t
\]

(1)

Were, \(\Delta Y_t\) is the first difference operator \((Y_t - Y_{t-1})\), \(\alpha\) is the intercept, \(\beta t\) is the model tendency component, \(\gamma\) is the coefficient that allows the stationary test (if \(\gamma = 0\), \(Y\) has a unitary root), \(p\) is the number of lag terms to be included in the model and \(\varepsilon_t\) is the random error term or the stochastic disturbance.

The Phillips & Perron - PP test\(^5\) was also used to verify the presence or not of unitary root. The difference between both tests is that the Phillips-Perron test gives us the guarantee that the disturbances are not correlated and have constant variance. Opposed to the Augmented Dickey-Fuller test, the Phillips & Perron test does not include the lag difference terms, but may include the tendency and the intercept terms.

The KPSS (Kwiatkowski, Phillips, Schmidt & Shin) test\(^5\) was developed as a form to complement the analysis of the traditional unitary root tests, such as the ADF and PP tests. On the contrary of the ADF and PP tests, the KHPSS test considers as the null hypothesis that the series is stationary, or stationary around a deterministic tendency, against the existence of a random path as an alternative hypothesis.

3.2 Normality Tests: Jarque-Bera (JB)

The Jarque-Bera (JB) normality test is based on the difference between the asymmetric and kurtosis of the series and normal law coefficients, used to test the null hypothesis that the sample comes from a normal distribution. To conduct this test, it is necessary to first calculate the asymmetry and kurtosis of the residuals using the test statistic:

\[
JB = n \left[ \frac{S^2}{6} + \frac{(C - 3)^2}{24} \right]
\]

(2)

were, \(JB\) is the Jarque-Bera test, \(S\) is the symmetric coefficient of the observations, \(C\) is the kurtosis

\(^3\) See Dickey & Fuller (2007).


\(^5\) See Kwiatkowski et al. (1992).
coefficients of the observations and $n$ is the number of observations. Assuming the normally null hypothesis, the JB statistic follows a chi-square distribution with two degrees of freedom. If the JB value is too low, the random error normal distribution cannot be rejected. However, if the JB value is too high, the normal distribution behavior of the random error or residuals is rejected. If the calculated $p$ chi-square statistic value is sufficiently low, the hypothesis of the residuals having a normal distribution can be rejected. If the $p$ value is high, the normally hypothesis is accepted.

3.3 The ARFIMA-FIGARCH Model

The ARFIMA Model $(p,d,q)$ is a generalization of the ARIMA $(p,d,q)$ with fractional integrated $(d \in \mathbb{R})$, introduced by Granger & Joyeux (1980) and Hosking (1981) which formulation for a discrete time process $(y_t)$ is given by:

$$\Phi(L)(1-L)^d y_t = \Theta(L) \varepsilon_t^2,$$

(3)

Where, $d$ is the fractional integrated parameter, $\Phi(L)$ and $\Theta(L)$ is the lag operator polynomial of $p$ and $q$ order, respectively situated outside of the unitary circle and $\varepsilon_t$ is the random error.

The difference between the ARFIMA Model and the ARMA Model is that the decay of the auto regression function of the first model is slow, while the decay of the second is fast and exponential. Hosking (1981) demonstrated that the ARFIMA $(p,d,q)$ process given by equation (3) is: i) stationary, if $d < 0.5$ and if all roots of $\Phi(L) = 0$ are out of the unitary circle; ii) invertible, if $d > -0.5$ and all roots of $\Theta(L) = 0$ are out of the unitary circle.

The auto regression function process decays more slowly than geometrically for any $d$ different from zero. According to Hosking (1981), when $-0.5 < d < 0.5$, the series is stationary and invertible. If $d = 0$, the process has a stationary error with short memory. When $d = 1$, the process has a unitary root with infinite variance. If $d = -0.5$, then the series is stationary, but not invertible. However, if $0 < d < 0.5$, then the autocorrelations are positive and the autocorrelation function decreases monotonically and hyperbolically towards zero. The correlation among far away observations can be relatively high, which implies the existence of long run memory. If $0.5 < d < 1$, the process still has long memory, but with infinite variance. Hence, if $0.5 < d < 0$, then the process has short memory and is anti-persistent. Finally, processes with $-1 < d < -0.5$ are atypical for economic data.

A new class of models proposed by Baille, Bollerslev & Mikkelsen (1996) called FIGARCH $(p,d,q)$ - Fractional Integrated GARCH, is defined as:

$$\Phi(L)(1-L)^d \varepsilon_t^2 = \omega + \left[1 - \beta(L)\right]\varepsilon_t^2 - \sigma_t^2$$

(4)

Where, $0 \leq d \leq 1$ designates the parameter of fractional integration with the $\Phi(L)$ and $[1-\beta(L)]$ roots located out of the unitary circle, so that the stationarity of the covariance can be guaranteed. Hence, re-ordering the terms of equation (4):

$$\left[1 - \beta(L)\right]\sigma_t^2 = \omega + \left[1 - \beta(L)\right] \phi(L)(1-L)^d \varepsilon_t^2,$$

(5)

So that the conditional variance of $\varepsilon_t^2$ can be re-written as:

$$\sigma_t^2 = \omega \left[1-\beta(L)\right]^{-1} + \phi(L)(1-L)^d \frac{\sigma_t^2}{\varepsilon_t^2},$$

(6)

$$\sigma_t^2 \equiv \omega \left[1-\beta(L)\right]^{-1} + \lambda(L) \sigma_t^2,$$

(7)

Where $\lambda(L) = \lambda_1 + \lambda_2 L^2 + \ldots.$

The parameter $d$ works as an indicator of the shock velocity propagation, that is, the greater its value, the greater will be the persistence effect of the conditional variance.

The used estimation will occur by means of simultaneously obtaining the parameters of the ARFIMA–FIGARCH model. According to Bayer (2008), the tendency of recent studies that demonstrates the advantage of the simultaneous estimation of the models’ coefficients. The parameters were estimated using the Maximum Likelihood Method.
3.4 Error Distribution

According to previous presentation in item 2.3, for each model: Gaussian (Normal), t-Student and Generalized Error Distribution (GED) distributions were adjusted, as described below by the log-likelihood function:

**Normal Distribution**

\[
L_{\text{normal}} = -\frac{1}{2} \sum_{i=1}^{n} \left[ \ln(2\pi) + \ln(\sigma_i^2) + \varepsilon_i^2 \right] \quad (8)
\]

**t- Student Distribution**

\[
L_{\text{student-}t} = \ln \left[ \Gamma \left( \frac{v+1}{2} \right) \right] - \ln \left[ \Gamma \left( \frac{v}{2} \right) \right] - 0.5 \ln[v(v-2)] - 0.5 \sum_{i=1}^{n} \left[ \ln \sigma_i^2 + (1+v) \ln \left( 1 + \frac{\varepsilon_i^2}{v-2} \right) \right] \quad (9)
\]

Where, \( \Gamma(\cdot) \) is a gamma function, \( v \) corresponds to the degrees of freedom. Hence, \( v > 2 \), if \( v \to \infty \), the t-student distribution converges to a normal distribution.

**t- Student Distribution Asymmetric**

\[
L_{t-\text{skew}} = \left\{ \begin{array}{ll}
\ln \Gamma \left( \frac{v+1}{2} \right) - \ln \Gamma \left( \frac{v}{2} \right) - 0.5 \ln[v(v-2)] + \\
+ \ln \left( \frac{2}{\xi + 1} \right) + \ln(\xi) \\
-0.5 \sum_{i=1}^{n} \left[ \ln \sigma_i^2 + (1+v) \ln \left( 1 + \frac{(se_i + m)^2}{v-2} \xi^{2} \right) \right] \end{array} \right\} 
\]

(10)

Where \( I_i = \begin{cases} 
1 & \text{se } \xi_i \geq - \frac{m}{s} \\
-1 & \text{se } \xi_i < - \frac{m}{s}
\end{cases} \)

\( \xi \) is the asymmetric parameter

\( v \) is the degree of freedom

\[
m = \frac{\Gamma \left( \frac{v+1}{2} \right) \sqrt{v-2}}{\pi \Gamma \left( \frac{v}{2} \right)} \left( \xi - \frac{1}{\xi} \right) e
\]

\[
s = \sqrt{\frac{s^2 + 1}{\xi^2} - 1} - m^2
\]

3.5 Data

The data used in this paper are daily price quotations of fat ox. The prices are given in R$ by arroba\(^6\), for the January 02, 2000 to December 30, 2012 period, amounting a total of 1242 observations. The data source is the Centro de Estudos Avançados em Economia Aplicada (CEPEA-ESALQ/USP) of the University of São Paulo.

4. EMPIRICAL RESULTS AND ANALYSIS

4.1 Graphic Analysis and Preliminary Tests

The daily returns were calculated using the formula: \( r_t = \ln(P_t) - \ln(P_{t-1}) \). Where \( P_t \) represents the fat ox prices of the day \( t \) and \( P_{t-1} \) the fat ox price of the previous day \( (t - 1) \). Figures 1 and 2 show the behavior of price quotation and fat ox daily returns series in the State of São Paulo in the period analyzed.

A prominent volatility of returns can be noted, upon a visual inspection of Figure 2 in the considered period, with the presence of outliers mainly in the years of 2009 and 2012. Therefore, in order to use the ARFIMA-FIGARCH models it was necessary to test the normality and stationarity of the fat ox daily price returns.

Some of the basic descriptive statistics are presented in Table 1. It can be observed that the daily price returns of fat ox presented a leptocúrtica distribution due to an excess of kurtosis (6.267754) in relation to the normal distribution (3.0). The Jarque-Bera statistic indicates the rejection of

\(^6\) An arroba is equivalent to 15 kg.
normality distribution of the series, with the p-value equal to zero.

The Q – Q Plot represents one of the most used graphic methods in the verification of the normality of time series. The used procedure consists in a graphic comparison of the theoretical quantile of the normal distribution with the quantile of the sample data. Figure 3 shows the existence of a non-linear relation among the theoretical and empirical quantile, very prominent in the distribution tails, indicating heavier tails in the empirical tail. Hence, all tests rejected the normality hypothesis of the analyzed series.

As seen in Table 2, the Augmented Dickey-Fuller - ADF (1979), Phillips-Perron - PP (1988) and Kwiatkowski, Phillips, Schmidt & Shin - KPSS tests with constant and tendency, show that the fat ox return series are stationary with no unitary roots.

4.2 ARFIMA-FIGARCH Models

4.2.1 Model selection criteria among error distributions

After the stationarity confirmation, the ARFIMA-FIGARCH models were chosen for the estimation of the equation mean and the series variance of the fat ox price returns, in order to eliminate serial correlation problem. Twelve models were calibrated using three distributions types for the residuals: normal (Gaussian), student’s t and asymmetric student’s t.

The black underline models in Table 3 were the ones with best prediction results. An important characteristic of the analysis was that the models that considered a conditional distribution different from the normal (Gaussian) presented better results. The Student’s t Asymmetric distribution was the best adjusted one. Among the analyzed models ARFIMA (1.d.0) - FIGARCH (1.d.0) model, based on the Akaike (AIC), Schwartz (SBC) and log-Likelihood criteria, was the one that best adjusted to series behavior.

The software used to estimate the data and models regression was the G@RCH package in the Ox language.

The estimates for the ARFIMA (1.d.0) – FIGARCH (1.d.0) models are presented in Table 4. With respect to the quality the adjustments of the models, the conditional heteroskedasticity could be eliminated, since the p-values were above 0.05 accepting the homoscedasticity null hypothesis, i.e., the residuals as well as their squares are not auto correlated.

Observing the results of the ARFIMA (1.d.0) – FIGARCH (1.d.0) Asymmetric Student’s t selected model, it can be said that the fat ox price volatility can be described as a fractional long-memory process, indicating persistence and a hyperbolic decay, with a stationary (reversible) character in the mean as well as in the variance.

The fractional coefficients (d) of the mean and conditional variance were significant, rejecting the null hypothesis of the parameters equal to zero.

5. CONCLUSIONS

The utilization of models to represent a long-memory fractional process was essential for the fat ox price volatility analysis in the State of São Paulo. In general, studies related to market channels are rare in view of the complexity of the macroeconomic aspects involved. The models used in this paper presented a strong methodological advance in the treatment of fat ox price returns, serving as important tools in risk management by investors.

Based on the Akaike, Schwartz and log-Likelihood criteria’s the chosen model was the asymmetric ARFIMA (1.d.0) - FIGARCH (1.d.0) - student t. The results showed that the fat ox price volatility describes a long-memory factionary process, indicating persistence and a slow hyperbolic decay, however with a stationary (reversible) character in the mean as well as in the variance. The conditional mean and variance fractional coefficients (d) were significant at the 5% level.

The everyday fat ox market panorama presents strong evidence corroborating with the results of this paper, since a shock in supply yields increases in the commodity price levels.

Finally, the empirical results of this paper suggest the utilization of the proper strategic instruments of
hedging in view of the accentuated shock persistence in the fat ox price volatility returns.

6. REFERENCES


Figure 1 – Daily cotation of fat ox price (in R$/@).

Figure 2 – Daily returns of fat ox price (in R$/@).

Table 1 – Fat ox price returns statistic summary.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>Median</th>
<th>Maxim</th>
<th>Minimum</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.000255</td>
<td>0.000203</td>
<td>0.028137</td>
<td>-0.022263</td>
<td>0.004831</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Asymmetry</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>p-value</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.216841</td>
<td>6.267754</td>
<td>562.3309</td>
<td>0.000000</td>
<td>1242</td>
</tr>
</tbody>
</table>

Figure 3 – Fat ox Q–Q plot daily returns.

Table 2 – Stationary tests for the fat ox price return series.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF</th>
<th>Critical Value (5%)</th>
<th>PP</th>
<th>Critical Value (5%)</th>
<th>KPSS</th>
<th>Critical Value (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat Ox</td>
<td>-12.1148</td>
<td>-3.4134</td>
<td>-32.0845</td>
<td>-3.4134</td>
<td>0.0889</td>
<td>0.1460</td>
</tr>
</tbody>
</table>
Table 3 – Error distribution selection criteria - different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Error Distribution</th>
<th>AIC</th>
<th>SBC</th>
<th>Likelihood-log</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARFIMA (0.d.1) - FIGARCH</td>
<td>Normal</td>
<td>-8.0967</td>
<td>-8.0720</td>
<td>5034.09</td>
</tr>
<tr>
<td>(1.d.0)</td>
<td>Student t</td>
<td>-8.1165</td>
<td>-8.1454</td>
<td>5065.29</td>
</tr>
<tr>
<td></td>
<td>Asymmetric Student t</td>
<td>-8.1457</td>
<td>-8.1467</td>
<td>5066.50</td>
</tr>
<tr>
<td>ARFIMA (1.d.0) - FIGARCH</td>
<td>Normal</td>
<td>-8.0967</td>
<td>-8.0719</td>
<td>5034.30</td>
</tr>
<tr>
<td>(1.d.0) (*)</td>
<td>Student t</td>
<td>-8.1455</td>
<td>-8.1166</td>
<td>5065.44</td>
</tr>
<tr>
<td></td>
<td>Asymmetric Student t</td>
<td>-8.1458 *</td>
<td>-8.1468 *</td>
<td>5066.68 *</td>
</tr>
<tr>
<td>ARFIMA (2.d.0) - FIGARCH</td>
<td>Normal</td>
<td>-8.0946</td>
<td>-8.0616</td>
<td>5034.71</td>
</tr>
<tr>
<td>(1.d.1)</td>
<td>Student t</td>
<td>-8.1420</td>
<td>-8.1049</td>
<td>5061.21</td>
</tr>
<tr>
<td></td>
<td>Asymmetric Student t</td>
<td>-8.1424</td>
<td>-8.1112</td>
<td>5066.45</td>
</tr>
<tr>
<td>ARFIMA (0.d.2) - FIGARCH</td>
<td>Normal</td>
<td>-8.0944</td>
<td>-8.0614</td>
<td>5034.61</td>
</tr>
<tr>
<td>(1.d.1)</td>
<td>Student t</td>
<td>-8.1415</td>
<td>-8.1044</td>
<td>5064.90</td>
</tr>
<tr>
<td></td>
<td>Asymmetric Student t</td>
<td>-8.1421</td>
<td>-8.1068</td>
<td>5066.22</td>
</tr>
</tbody>
</table>

Source: research outcome.

* Best model according to the chosen criteria

Table 4 – Results of the estimation of the ARFIMA-FIGARCH (1.d.0) (1.d.0) Model.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Normal</th>
<th>Student t</th>
<th>Asymmetric Student t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional Mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.000512 (0.3923)</td>
<td>0.000310 (0.0264)</td>
<td>0.000718 (0.0158)</td>
</tr>
<tr>
<td>AR (1)</td>
<td>0.08126 (0.0106)</td>
<td>-0.06725 (0.0699)</td>
<td>-0.06567 (0.0815)</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.26503 (0.0000)</td>
<td>0.25132 (0.0000)</td>
<td>0.25370 (0.0000)</td>
</tr>
</tbody>
</table>

Conditional Variance

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Student t</th>
<th>Asymmetric Student t</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.09379</td>
<td>0.08769</td>
<td>0.09054</td>
</tr>
<tr>
<td>( d )</td>
<td>0.31681</td>
<td>0.28735</td>
<td>0.29618</td>
</tr>
<tr>
<td>( v )</td>
<td></td>
<td></td>
<td>5.05609</td>
</tr>
<tr>
<td>( \ln(d) )</td>
<td></td>
<td></td>
<td>0.05609</td>
</tr>
<tr>
<td>Log (L)</td>
<td>5034.30</td>
<td>5065.44</td>
<td>5066.68</td>
</tr>
<tr>
<td>AIC</td>
<td>-8.0967</td>
<td>-8.1455</td>
<td>-8.1458</td>
</tr>
<tr>
<td>SBC</td>
<td>-8.0719</td>
<td>-8.1166</td>
<td>-8.1468</td>
</tr>
<tr>
<td>Jarque-Bera (JB)</td>
<td>127.44</td>
<td>129.82</td>
<td>130.07</td>
</tr>
<tr>
<td>( Q(5) )</td>
<td>6.2040</td>
<td>7.2778</td>
<td>7.0475</td>
</tr>
<tr>
<td>( Q(10) )</td>
<td>15.9098</td>
<td>17.4110</td>
<td>17.1945</td>
</tr>
<tr>
<td>( Q(20) )</td>
<td>40.2884</td>
<td>42.2925</td>
<td>41.7521</td>
</tr>
<tr>
<td>( Q^2(5) )</td>
<td>5.8392</td>
<td>6.1897</td>
<td>6.0706</td>
</tr>
<tr>
<td>( Q^2(10) )</td>
<td>7.4594</td>
<td>7.6116</td>
<td>7.6017</td>
</tr>
<tr>
<td>( Q^2(20) )</td>
<td>14.6706</td>
<td>14.5252</td>
<td>14.5445</td>
</tr>
</tbody>
</table>

The numbers between parenthesis are the probability values (p-value), at a 5% significant level.

Source: Research outcome.